3.2 Introduction to Polynomial Functions

Polynomial Function

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

$a_n, a_{n-1}, \ldots, a_2, a_1, a_0 \Longrightarrow$ **real numbers**

$n \Longrightarrow$ a whole number

The **degree of** $f(x)$ is the highest exponent "n"

The **leading coefficient**, a_n, is the coefficient of the variable to the highest power.

When graphed, polynomial functions of degree 2 or higher have graphs that are smooth and continuous. There are no sharp corners or no breaks (no gaps), and can be drawn without lifting your pencil.

❖ End Behavior of Polynomial Functions

As x approaches ∞ or $-\infty$, f eventually becomes forever increasing or decreasing.

The Leading Term (Coefficient) Test

$a_n =$ the leading coefficient

$n =$ the degree of the polynomial

	$a_n > 0$	$a_n < 0$
n Even Degree	Left End: up Right End: up	Left End: down Right End: down
n Odd Degree	Left End: down Right End: up	Left End: up Right End: down

Ex. Use the Leading Term Test to determine the end behavior of the following:

(a) $f(x) = -2x^4 + 2x^3$

(b) $f(x) = \dfrac{7}{6}x(x-9)^3(x+4)^2(3x-5)$

❖ Zeros of Polynomial Functions

Zeros of Polynomial: the values of x for which $f(x) = 0$

Solutions $\Longleftrightarrow$ x-values $\Longleftrightarrow$ x-intercepts $\Longleftrightarrow$ Zeros $\Longleftrightarrow$ Roots

❖ **Multiplicities of Zeros**

After **factoring** the equation for the polynomial function *f*, if the same factor $(x-r)$ occurs *k* times, we call *r* a **zero with multiplicity *k***.

Ex. $(x-5)^2$

The zero is 5 with multiplicity 2.

Ex. $(x+4)^3$

The zero is ______ with multiplicity ______.

Even Multiplicity

If the multiplicity of a zero is an **EVEN** number, the graph will **TOUCH** the *x*-axis.

Odd Multiplicity

If the multiplicity of a zero is an **ODD** number, the graph will **CROSS** the *x*-axis.

Strategy for Graphing Polynomial Functions

1.) **Find *x*-intercepts:** mark all the zeros on the *x*-axis.
2.) **Determine the end behavior:** use the end behavior on the left-most and the right-most *x*-axis
3.) **Determine the multiplicity:** for each *x*-intercept, use TOUCH or CROSS to fill in the graph.
4.) **Find the *y*-intercept:** compute $f(0)$.

Ex. Use knowledge of end behavior and multiplicity of zero to sketch the function:

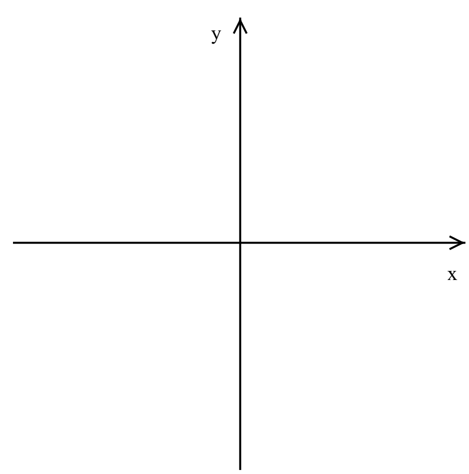
$f(x)=-2x^3(x+2)^2(x+1)$

Degree of the Poly.: ______

Leading Coeff.: _________

y-intercept: _____________

Zero	Multiplicity	End Behavior



Ex. Use knowledge of end behavior and multiplicity of zero to sketch the function:

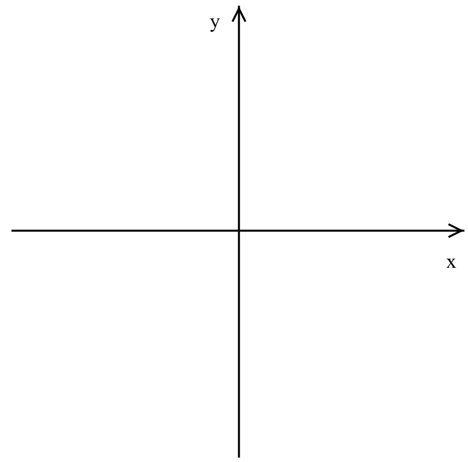
$f(x) = x^3 + 2x^2 - x - 2$

Degree of the Poly.: ______

Leading Coeff.: _________

y-intercept: _____________

Zero	Multiplicity	End Behavior



If *f* is a polynomial function of degree *n*, then the graph of *f* has at most $n-1$ **turning points**.

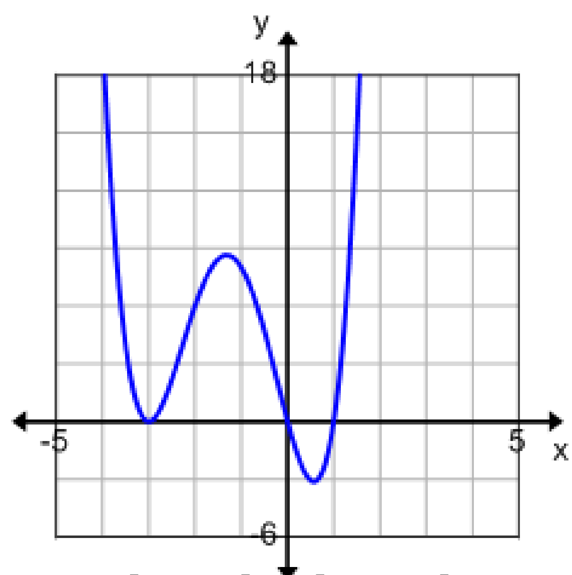
→ Use the fact that the maximum number of turning points of the graph is $n-1$ to check whether it is drawn correctly.

Ex. Determine if the graph can represent a polynomial function. If so, assume that the end behavior and all turning points are represented in the graph.

(a) Determine the minimum degree of the polynomial based on the number of turning points.

(b) Determine whether the leading coefficient is positive or negative based on the end behavior and whether the degree of the polynomial is odd or even.

(c) Approximate the real zeros of the function, and determine if their multiplicities are even or odd.



$[-5, 5, 1]$ by $[-6, 18, 1]$